Griffiths

5.11. Energy gap between $2S_{1/2}$, $2P_{1/2}$ due to lamb shift.

$$k(n,0) \quad vs. \quad \left\{ k(n,l) \pm \frac{1}{\pi (j+\frac{1}{2})(l+\frac{1}{2})} \right\}.$$

$$= 13.7 \quad vs. \quad 0.05 \pm \frac{1}{\pi(3/2)} \approx -0.15$$

$\Rightarrow$ The energy split is approximately $\frac{\alpha^5 mc^2 (13)}{4n^3}$.

$$\approx \frac{1}{(137)^5} \frac{(0.5\times10^6 eV)(13)}{32}$$

$$\approx 2\times10^{-11} \times 0.2\times10^6 eV \approx \boxed{4\times10^{-6} eV}.$$

$E = hf$, $h = 4.14\times10^{-15} eVs$.

$$f = \frac{4\times10^{-6} eV}{4.14\times10^{-15} eVs} \approx 10^9 s^{-1} \approx 10^3 \times MHz.$$

This is consistent with experiment data.